6TH GRADERS UNDERSTANDING OF MULTIPLICATIVE STRUCTURES: A 5 YEAR FOLLOW UP STUDY

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An earlier study analysed responses by 70 young children to a variety of multiplication and division word problems at four interview stages in a 2-year longitudinal study (Mulligan, 1992). 75% of the children were able to solve the problems using a wide variety of counting, grouping and modelling strategies and the semantic structure of the problem strongly influenced solution process. A follow up study was conducted ***with the same sample*** *(n=45) in Grade 6, prior to entry to secondary school. Identical problem structures were used including problems involving decimals. A similar pattern of performance to the longitudinal study revealed a 60-85% success rate except for Sub-division, and poorer results for decimal problems. Informal methods used by children in the early stages of the study dominated their understanding of multiplicative structures in Grade 6. Evidence of children's lack of conceptual understanding of multiplication and division was revealed.*

INFORMAL MATHEMATICS

Research investigating young children's understanding of number concepts and problem-solving processes has revealed widespread use of informal or intuitive strategies developed prior to formal instruction (Carpenter and Moser 1984; Hughes, 1986; Steffe, Cobb and Richards, 1988; Steffe and Wood, 1990). Studies investigating solutions to addition and subtraction problems have indicated that children use a wide variety of modelling and counting strategies that reflect the semantic structure of the problem (Carpenter and Moser, 1984; De Corte and Verschaffel, 1987). These studies indicate that children possess considerable mathematical knowledge and skills prior to formal instruction that is developed from their informal experiences. However, when children experience formal instruction it cannot be assumed that their conceptualisations are linked with formal mathematical ideas or their own strategies match those encouraged in instruction (Hiebert, 1984; 1990). While the use of informal strategies often provides a meaningful way for solving problems initially, these strategies may be ineffectual for more complex mathematical tasks.

Many researchers have expressed concern over the widespread and continued use of informal methods well into secondary school (Bell, Swan and Taylor, 1981; Booth, 1984; Fischbein et al., 1985; Hart, 1981). These informal methods, consisting largely of counting, adding-on or building-up strategies, may be adequate for simple items but are not generalisable to more complex mathematical situations or operations, e.g. fractions or applications of algebra. Researchers investigating the strategies secondary school children used when solving a problem found "that many children are not using the proper mathematical methods taught them at school but rather are relying upon naive intuitive strategies" (Booth, 1981).

RESEARCH ON MULTIPLICATION AND DIVISION PROBLEMS

Over the past decade, researchers have analysed secondary student's solution processes to multiplication and division word problems based on differences in semantic structure, mathematical structure, size of quantities used, and student's intuitive models (Bell, Fischbein and Greer, 1984; Bell, Greer, Grimison and Mangan, 1989; Brown, 1981; De Corte, Verschaffel and Van Coillie, 1988; Fischbein et al., 1985; Nesher, 1988; Vergnaud, 1988). Various studies have shown that children's difficulties in solving problems are related to the size and complexity of numbers in the problem (Brown, 1981; Bell et al.,1981; Bell et al., 1984; Bell et al., 1989). When children were presented with problems having the same mathematical, semantic, and surface structure but differing in terms of the nature of the given numbers, they changed their minds or were unable to decide about the operation to be used.

The 'multiplier effect' has been identified where increases in problem difficulty occur when the multiplier changes from an integer to a decimal number, and to a decimal number less than 1. The choice of operation in solving a multiplication or division word problem could also be influenced by the notion 'multiplication makes bigger' and 'division makes smaller'. A 'competing-claims' theory (Bell et al., 1989), in relation to choice of operation in multiplication and division word problems suggests that children optimize the possibly competing claims of four factors: (i) numerical preferences, (ii) conformity with the lowest level of problem structure, (iii) numerical misconceptions, and (iv) ordering the quantities. Furthermore, hierarchies of difficulty in the word problems were found through to adulthood showing that students relied on methods causing "less cognitive strain before adopting harder ones" (p.447).

More recently, a growing number of studies on young children's solution strategies to multiplication and division problems have emerged (Anghileri, 1989; Boero, Ferrari and Ferrero, 1989; Brekke and Bell, 1992; Kouba, 1989; Mulligan, 1991; 1992; Murray, Olivier and Human, 1992; Steffe, 1988). These studies have provided complementary evidence that the semantic structure of the problem, an understanding of the problem context, and the development of counting, grouping and addition strategies influence solution process. Underlying intuitive models for multiplication and division may also be attributed to children's informal development of these processes and the difficulties experienced when problem-solving operations come into conflict with constraints of these models (Boero et al.,1989; Fischbein, 1989; Mulligan, 1991).

METHODOLOGY

A follow up study was conducted to investigate 6th Grade children's understanding of multiplication and division through word problems involving simple number facts, their solutions to problems involving decimals in measurement contexts, and their ability to generate written problems from symbolic forms. The study aims to further map the development of multiplication and division processes and identify key factors in this process. Comparisons between the children's performance, strategy use and intuitive models shown in Grades 2-3 and Grade 6 were analysed by individual profiles. Clinical interviews were conducted using identical procedures to those employed in the longitudinal study. The interview sample retained 45 of the 60 girls previously interviewed in Grade 3 in the longitudinal study (Mulligan, 1992). The girls were interviewed in December of Grade 6 prior to entry to secondary school and they were permitted to record their mathematical ideas on paper. Each interview was approximately one hour in duration.

INTERVIEW TASKS

The girls were required to initially solve ten multiplication and division word problems (Table 1) representing each problem structure used in the longitudinal study. Number size was increased to include more difficult combinations using number triples (8,9,72), (15,6,90), (7,8,56), (9,12,108), (12,3,36), (6,7,42), (15,5,75), (7,12,84), (24,6,4) respectively but these were not much more difficult than the large number combinations used in the previous interview in Grade 3.

Multiplication	Division
Repeated Addition There are 8 children in the library and 9 children are seated at each table. How many children are there altogether in the library?	Partition (Sharing) $72.00 was shared equally between 9 children. How much money did they get each ?
Rate If you need 15c to buy one sticker how much money do you need to buy 6 stickers?	Rate Peter bought 15 folders for $75.00. If each folder cost the same price how much did one folder cost? How much did 7 folders cost?
Factor John has 7 books on his shelf and Sue has 8 times as many. How many books does Sue have?	Factor Simone has 42 books on her shelf and this is 6 times as many as Lisa. How many books does Lisa have ?
Array There are 9 lines of children at assembly with 12 children in each line. How many children are there altogether ?	Quotition 84 cards were shared equally between the children. If they each had 12 cards, how many children were there?
Cartesian Product There are 12 different ice-cream flavours and 3 different sized cones. How many different choices of ice-cream cones could you make?	Sub-division I have 4 large chocolate bars to be shared evenly between 24 children. How much chocolate will each child get?

Four problems using multiplication of whole numbers and decimals in measurement contexts were included (Table 2). Additional tasks involving algorithms and children's writing of word problems to match symbolic forms were given, but are not reported in this paper.

Table 2: **Multiplication of Decimals Problems**

Repeated Addition (length) Susan used 3 epics of ribbon each 4.6 metres long. How much ribbon did she use altogether? **Rate (speed)** A girl walked at an average speed of 4 kilometres per hour for 3.2 hours. How far did she walk?	**Factor (mixture)** A painter mixes a special colour by using 3.2 times as much red as yellow paint. How much red paint should the painter use with 4 litres of yellow paint? **Enlargement (factor/size)** A photograph is made bigger (enlarged) 3 times its size. If the length of the original photograph is 5.3 cm, what will be the length of the big photograph?

DISCUSSION OF RESULTS

Performance across the ten problem structures indicated in Table 3 revealed a 60-85% success rate with few differences found between the performance on multiplication and division problems. There was a marked improvement in performance on the Rate, Cartesian Product and Factor problems in comparison with results in Grades 2 and 3. However, performance on the Sub-division problem was very poor possibly due to the increase in difficulty of the number combinations used where children were required to divide 4 chocolates by 24 to gain 6 equal parts. They were unable to simply halve and rehalve (or double) the quantities as for the earlier interview.

Performance across many problem structures was *generally lower than in Grade 3 and much lower than expected for children entering secondary school.* Although number combinations were slightly more difficult, basic number fact knowledge at this level would be required for coping with a range of multiplicative problems. A preliminary analysis of solution strategies revealed widespread use of known and derived facts but counting and grouping methods previously used in Grades 2-3 were also used. *Further analysis of individual profiles showed that 32 % of the sample were unable to solve half of the initial ten word problems.* In some cases *performance had decreased since the earlier interviews* with evidence that the children were relying on rote learned multiplication tables that caused frequent error and no estimation of the answer. It appears that many of these children had stopped analysing the problems and focussed on manipulating numerical combinations. Use of immature and inadequate additive procedures dominated responses for the majority of these children.

Table 3 Percentage of Correct Responses by Problem Structure and Grade Level (Large Number Problems)

PROBLEM STRUCTURE	Grade		
	2* **n = 70**	**3** **n = 60**	**6** **n = 45**
Multiplication			
1. Repeated Addition	27	80	81
2. Rate	72	98	86
3. Factor	0	47	67
4. Array	39	78	76
5. Cartesian Product	1	10	64
Division			
6. Partition	34	83	79
7. Rate	51	85	79
8. Factor	0	10	60
9. Quotition	34	73	67
10. Sub-division	10	43	26
Multiplication of Decimals			
Repeated Addition (4.6 x 3)			62
Rate (4 x 3.2)			43
Factor (3.2 x 4)			50
Enlargement (3 x 5.3)			81

* March Interview

Results for the four problems involving multiplication of a decimal number >1 revealed common difficulties associated with influence of semantic structure particularly with the Rate and Factor problems. However, difficulties with operating on decimals were revealed but the full impact of the 'multiplier effect' was not possible without using decimals less than 1. Conceptual misunderstandings related to decimal notation and operations on decimals were revealed in many of the responses even where the solution was correct. Few children were able to provide reasonable estimates of the solutions.

IMPLICATIONS FOR TEACHING

The 5 Year follow up study has provided valuable insights into the development of multiplication and division processes and the difficulties experienced by a significant proportion of children. On the basis of children's responses, the use of additive and estimation strategies, especially efficient use of multiple and group counting, might be a more effective way, initially, of teaching multiplication and division. The identification of young children unable to use these strategies and solve simple word problems is essential so that the progression to effective use of known facts will follow.

Teaching programs could incorporate the development of informal strategies rather than focussing only on mastering number facts and computational skills too early, that may not relate to the child's strategy development. Teachers could facilitate more meaningful learning by establishing links between children's intuitive strategies and the formal teaching of addition, subtraction, multiplication and division. Perhaps the teaching of these processes in an integrated fashion, and based on the child's experience of a range of related problem situations might best reflect the natural development of these processes. The relative difficulty of different problem structures and number combinations has been more clearly identified and thus, teachers could expose children to these with a better understanding of the relative ease or difficulty which children may encounter.

The difficulties experienced with operating on decimals warrants teaching emphasis on representations of decimals, solving a wide range of problem situations, attention to misconceptions about 'multiplication makes bigger', 'division makes smaller' and efficient counting, number fact and estimation strategies. Teachers may need to identify which 'competing' factors influence children's responses to multiplication and division tasks.

REFERENCES

Anghileri, J. (1989). An investigation of young children's understanding of multiplication. *Educational Studies in Mathematics,* 20 (4), 367-385.

Bell, A., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: the effects of number size, problem structure and context. *Educational Studies in Mathematics,* 15, 129-147.

Bell, A., Greer, B., Grimison L., & Mangan, C. (1989). Children's performance on multiplicative word problems : Elements of a descriptive theory. *Journal for Research in Mathematics Education,* 20 (5), 434 - 449.

Bell, A., Swan, M.& Taylor, G. (1981). Choice of operation in verbal problems with decimals. *Educational Studies in Mathematics,* 12, 399-420.

Boero, P., Ferrari, P., & Ferrero, E. (1989). Division problems: Meanings & procedures in the transition to a written algorithm. *For the Learning of Mathematics,* 9 (3), 17-25.

Booth, L. (1981). Child-methods in secondary mathematics. *Educational Studies in Mathematics,* 12, 29-41.

Booth, L. (1984) *Algebra: Children's strategies and Errors.* Windsor: NFER-Nelson

Brekke, G., and Bell, A.: 1992, 'Multiplicative structures at ages seven to eleven', In W. Geeslin and K. Graham (ed.), Proceedings of the Sixteenth Conference of the International Group for the Psychology of Mathematics Education, Program Committee of the 16th PME Conference, New Hampshire, USA, 1, 97-105.

Brown, M.L. (1981). Number operations. In K.M.Hart (Ed.), *Children Understanding Mathematics : 11-16.* London: John Murray.

Carpenter, T.P., & Moser, J.M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education,* 15 (3), 179-203.

De Corte, E. & Verschaffel, L. (1987). The effect of semantic structure on first grader's strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education,* 18 (5), 363-381.

De Corte, E., Verschaffel, L., & Van Coillie, V. (1988). Influence of problem structure, and response mode on children's solutions of multiplication word problems. *Journal of Mathematical Behaviour,* 7,97-216.

Fischbein, E., Deri, M., Nello, M.S., & Merino, M.S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education,* 16 (1), 3-17.

Fischbein, E. (1989). Tacit Models and Mathematical Reasoning', *For the Learning of Mathematics,* 9, (2), 9-14.

Hart, K.M. (Ed.). (1981). *Children's understanding of mathematics: 11-16.* London: John Murray.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal,* 5, 497-513.

Hiebert, J. (1990). The role of routine procedures in the development of mathematical competence. In T.J. Cooney & C.R. Hirsch (Eds.), *Teaching and learning mathematics in the 1990's,* 31-41, Reston, Virginia: N.C.T.M.

Hughes, M. (1986). *Children and number.* Oxford: Basil Blackwell.

Kouba, V.L. (1989). Children's solution strategies for equivalent set multiplication and division word problems. *Journal for Research in Mathematics Education,* 20 (2), 147-158.

Mulligan, J.T. (1991) The role of intuitive models in young children's solutions to multiplication and division word problems. Paper presented to the 15th Annual Conference of the Mathematics Education Research Group of Australasia, Perth.

Mulligan, J.T. (1992). Children's solutions to multiplication and division word problems: a longitudinal study, *Mathematics Education Research Journal*, 4, (1), 24-42

Murray, H., Olivier, A., and Human, P.: 1992, 'The development of young children's division strategies', In W. Geeslin and K. Graham (ed.), Proceedings of the Sixteenth Conference of the International Group for the Psychology of Mathematics Education, Program Committee of the 16 th Annual Conference, New Hampshire, USA, 2, 252-260.

Nesher, P. (1988). Multiplicative school word problems: Theoretical approaches and empirical findings. In J. Hiebert & M. Behr, (Eds.), *Number concepts and operations in the middle grades* , 2, 19-40, Hillsdale, N.J: Erlbaum.

Steffe, L. P. (1988). Children's construction of number sequences and multiplying schemes. In J. Hiebert & M. Behr, (Eds.), *Number concepts and operations in the middle grades*, 2, 119-140, Reston, Virginia: Erlbaum/ NCTM.

Steffe, L.P., Cobb, P., & Richards, V. (1988). *Construction of arithmetic meanings and strategies*. New York: Springer-Verlag.

Steffe, L.P., & Wood, T. (1990). *Transforming children's mathematics education: International Perspectives.* Hillsdale, New Jersey: Erlbaum.

Vergnaud, G. (1988). Multiplicative Structures. In J. Hiebert & M.Behr (Eds.), *Number concepts and operations in the middle grades.* 2, 141-161, Reston,Virginia: Erlbaum/N.C.T.M.